LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93940

NPS52-83-013

NAVAL POSTGRADUATE SCHOOL

Monterey, California

A COMMENTARY ON MILL'S LOGIC
BOOK I
OF NAMES AND PROPOSITIONS

Bruce J. MacLennan

October 1983

Approved for public release; distribution unlimited

Prepared for:

Chief of Naval Research
Arlington, VA 22217

FedDocs
D 208.14/2
NPS-52-83-013

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral J. J. Ekelund
Superintendent

D. A. Schrady
Provost

The work reported herein was supported in part by the Foundation Research Program of the Naval Postgraduate School with funds provided by the Chief of Naval Research.

Reproduction of all or part of this report is authorized.

This report was prepared by:

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NPS52-83-013
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): A Commentary on Mill's Logic Book I - Of Names and Propositions
5. TYPE OF REPORT & PERIOD COVERED: Technical Report
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Bruce J. MacLennan
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Postgraduate School, Monterey, CA 93943
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61152N: RR000-01-10; N0001483WR30104
11. CONTROLLING OFFICE NAME AND ADDRESS: Chief of Naval Research, Arlington, VA 22217
12. REPORT DATE: October 1983
13. NUMBER OF PAGES: 64
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Mill's Logic, J. S. Mill, epistemology, philosophy of science, scientific method, logic, propositions, definitions, universals, intension, extension, connotation, denotation.
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

Mill's Logic is the cornerstone of scientific method; yet, aside from Mill's Methods of Induction, its contents are not well known. This report attempts to make Book I of Mill's Logic more accessible to students of science and the philosophy of science. Each section of Mill's work is summarized. Most sections also include comments that criticize Mill's position, or relate the topic to more recent developments in the philosophy of science.

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY CA 93943-5101

A COMMENTARY ON MILL'S LOGIC

Book I

Of Names and Propositions

Bruce J. MacLennan

CONTENTS

Preface 1

INTRODUCTION 2

I. OF NAMES AND PROPOSITIONS 4

1. Of the Necessity of Commencing with an Analysis of Language 4

2. Of Names 6

1. Names are Names of Things 6

2. General and Singular Names 6

3. Concrete and Abstract 8

4. Connotative and Non-Connotative 9

5. Positive and Negative 13

6. Relative and Absolute 16

3. Of the Things Denoted by Names 17

1. Necessity of an enumeration of Nameable Things 17

2. Feelings, or states of Consciousness 18

3. Feelings must be distinguished from their physical antecedents 18

4. Perceptions 19

5. Volitions and Actions 20

6. Substance and Attribute 21

7. Body 21

8. Mind 22

9. Qualities 22

10. Relations 23

11. Resemblance 23

12. Quantity 24

13. Attributes Concluded 25

14. Recapitulation 25

4. Of Propositions 28

1. Nature and office of the copula 28

2. Affirmative and Negative Propositions 28

3. Simple and Complex 28

4. Universal, Particular, and Singular 30

5. Of the Import of Propositions 33

1. Is Proposition a Relation Between Two Ideas? 33

2. Is a Proposition a Relation Between the Meanings of Two Names? 33

3. Is a Proposition an Expression of Class Membership? 34

4. What it Really Is 35

5. What it Is that Propositions Assert or Deny 36

6. Propositions with Abstract Terms 39

6. Of Propositions Merely Verbal 40

1. Essential and Accidental Propositions 40

2. Essential Propositions are Identical Propositions 40

3. Individuals Have No Essences 43

4. Real Propositions, How Distinguished from Verbal 43

5. Two Modes of Representing the Import of a Real Proposition 44

7. On Classification and the Predicables 45

1. Classification, How Connected with Naming 45

2. The Predicables 46

3. Genus and Species 47

4. Kinds Have a Real Existence in Nature 47

5. Differentia 49

6. Property 50

7. Accident 51

8. Of Definition 52

1. A Definition, What 52

2. What Names can be Defined? 53

3. Complete versus Incomplete Definitions 54

4. Complete Definitions versus Descriptions 54

5. Real Definitions versus Nominal Definitions 55

6. Mathematical Definitions 57

7. Definitions Grounded on Knowledge of Corresponding Things 58

II. REFERENCES 59

Preface

Mill's *Logic* is the cornerstone of scientific method. Yet, except for Mill's Methods of Induction, its contents are largely unknown. Although Mill's less important works on sociological and political topics are widely reprinted, it is often difficult to find a copy of his *Logic*. The present work attempts to make Mill's *magnum opus* more accessible to students of science and the philosophy of science.

Mill's *Logic* presents serious difficulties to the modern reader. The work is long and dwells on many issues whose importance has declined. Conversely, there have been many developments in the philosophy of science since Mill's time. These, of course, he can't discuss. Finally, Mill's *Logic* makes heavy use of terms which are no longer current. These characteristics all decrease its accessibility to students.

This work follows the same outline as Mill's *Logic*. In each section I have summarized Mill's major points. Most sections also include comments that criticize Mill's position, or relate the topic to more recent developments in the philosophy of science.

The main references we have used are Mill's *Logic* (Mill, 1843), Nagel's editing of Mill's work (Nagel, 1950), and Killick's *Student's Handbook to Mill's Logic* (Killick, 1909). The latter was a model for this work.

The preparation of this report was supported in part by the Office of Naval Research under contract number N00014-82-WR-20162.

INTRODUCTION

Summary: A good starting place for a definition of logic is Whately's (1868): Logic is the science and art of reasoning. As a science it studies the mental processes that must take place whenever we reason; as an art it lays down rules, based on this analysis, that must be followed if we are to reason correctly.

Logic is concerned with inferences, not *intuitive* truths. By the latter Mill means our direct sensations, whether of the external world or our own mental states. These intuitive truths are beyond doubt, and no science is required to establish their truth. Further, no science can make us more confident of them. However, we must be careful not to mistake very rapid inferences for these intuitive truths. For instance, judging the distance to something we see is a very rapid inference that must be learned.

Mill states, "The province of logic must be restricted to that portion of our knowledge which consists of inferences from truths previously known, whether those antecedent data be general propositions or particular observations and perceptions. Logic is not the science of belief, but the science of *proof or evidence*. Insofar as belief professes to be founded on proof, the office of logic is to supply a test for ascertaining whether or not the belief is well grounded."

Mill explains the relation of logic to the other sciences as follows: "Logic, however, is not the same thing with knowledge, though the field of logic is co-extensive with the field of knowledge. Logic is the common judge and arbiter of all particular investigations. It does not undertake to find evidence, but to determine whether it has been found." Thus logic is the science of science itself.

Comments: Mill's use of the term logic is much wider than is usual now. It is the science of inference, which includes both inference from generals to particulars, or *deduction*, and inference from particulars to generals, or *induction*. In contemporary usage, logic is taken to mean deductive logic, and usually symbolic logic at that. A

better term for what Mill calls 'logic' would be 'scientific method'.

Mill, in the tradition of the sensationalistic empiricists (Locke, Berkeley, and Hume) takes *sensations* as the starting point for knowledge. This is also the position of the logical empiricists and logical positivists (such as Mach and Schlick), who followed Mill. This view commits the very error that Mill has cautioned us against. The data that is directly given to us is not sensations, but *perceptions*, i.e., organized sensations. That is, the basis for knowledge is not classified objects, such as trees, since classification is a process of inference. Nor is the basis unorganized "patches of color." Rather we perceive organized, but unclassified entities. It is only by a later process of analysis that we abstract out the "patches of color" that the sensationalists say are the basis of knowledge. We must start our analysis with perceptions because the integration of sensations into perceptions is an automatic process performed by our sensory apparatus. Processes not under our control are not the proper province of logic (or any art).

I. OF NAMES AND PROPOSITIONS

1. Of the Necessity of Commencing with an Analysis of Language

Summary: Why should a study of scientific method be concerned with language? "Logic is a portion of the art of thinking; language is evidently, and by the admission of all philosophers, one of the principal instruments or helps of thought; and any imperfection in the instrument or in the mode of employing it is confessedly liable, still more than in almost any other art, to confuse and impede the process and destroy all ground of confidence in the result." An even more fundamental reason for studying language is that we need this study to examine a central topic in logic, the import of *propositions.*

This is a central topic because "Whatever can be an object of belief or even of disbelief must, when put into words, assume the form of a proposition." That is, subjects can be conceived but not believed; only propositions can be believed or disbelieved. A *proposition* is "discourse in which something is affirmed or denied of something." A proposition has three parts:

- A *predicate*, which is a name indicating what is ffirmed or denied;
- A *subject*, which is a name denoting the thing which the predicate is affirmed or denied of;
- A *copula* (link), which is the sign indicating whether there is affirmation or denial.

Comments: I can add little to this except to note that since Mill's time the study of language has virtually *replaced* the study of logic. Hence, in the following sections I will often emphasize the non-linguistic aspects of the subject.

At this point I will mention that it is a common fallacy that Aristotle's analysis of the form of propositions is inadequate, since it does not cover all cases. First, it is said that it excludes relations, such as *less than*. But, if we consider a proposition such as 'Two is less than three', it is clear that this fits the subject-copula-predicate form: 'Two'

is the subject, 'is' is the copula and 'less than three' is the predicate. Further examples will be discussed later.

2. Of Names

1. Names are Names of Things

Summary: Mill begins with Hobbes' definition of a name: "A name is a word taken at pleasure to serve for a mark which may raise in our mind a thought like to some thought we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had or had not before in his mind." Thus names serve both to identify our own thoughts and to communicate our thoughts to others.

This suggests the question, "Are words the names of things or of our ideas of things?" It seems most proper to consider names to be the names of things. For example, when we say 'The sun set' we intend to convey something about the sun not our idea of the sun. In fact, we have specific linguistic mechanisms for talking about our ideas, as when we say 'The idea of the sun entered my mind.' In other words, propositions don't just inform the hearer of certain conjunctions of ideas in our mind; they also inform the hearer about what we believe about the things in reality.

Comments: This interpretation is necessary if science is to be of any value to us. If the propositions of science are to be valuable to technology, and life in general, they must be propositions about the world, not just our ideas of the world.

2. General and Singular Names

Summary: They are many ways that words can be categorized. First we can distinguish:

- Predicable (categorematic) terms, which can be used alone, either as the subject or the predicate of a proposition.
- Non-predicable (syncategorematical) terms, which can only form the parts of other names.

Thus, predicable terms name things, while non-predicable terms do not. Prepositions

and adverbs are examples of non-predicable words.

Comments: This is mostly a syntactic distinction. For example, adverbs and adjectives denote concepts just as much as nouns, it is just that the syntax of our language prevents us from using them as subject or predicate. Instead of 'Heavy is a burden,' we must say 'Heavy things are a burden.' There is no essential difference in the propositions expressed.

Even prepositions and conjunctions have a conceptual meaning; it is syntactic limitations that always require them to be used with other words. There are some words that serve a syntactic function only; these are the only truly non-predicable words.

Summary: There are two broad classes of names: *individual* and *general*. An individual name "is a name which is only capable of being truly affirmed, in the same sense, of one thing." For example, proper names are individual names. Now, it would not be possible to give everything, real or imaginary of which we might have cause to think, an individual name. Hence, we give *general* names to broad classes of things (such as 'stone') and indicate the individual in which we are interested by phrases such as 'this stone' or 'the stone on the table.'

This is not the most important function of general names, however. "It is by their means that we are enabled to assert *general* propositions, to affirm or deny any predicate of an indefinite number of things at once." A general name is "a name which is capable of being truly affirmed, in the same sense, of each of an indefinite number of things."

It is necessary to distinguish general names from *collective* names. Collective names are really individual names in which the individual named is a composite entity made of other individuals. For example 'The US Navy' is a collective name for a particular collection of people; it is predicable of this collection as a whole, and only this collection; it is not predicable of the individual persons in this collection. On the other

hand, 'member of the US Navy' is a *general* name that is predicable of each individual in this collection and *not* of the collection as a whole.

Comments: General names are the most important class of names, because it is these names that denote *concepts* or *universals*. Scientific knowledge would not be very applicable if its principles and laws were only applicable to the individuals for which the laws and principles had been verified. If this were the case, scientific laws would only summarize the result of yesterday's experiments; they would not give us principles that can be applied tomorrow. Hence, scientific principles are expressed in terms of concepts or universals that subsume an indefinite number of individuals.

Certainly one of the major values of general names (and the concepts they name) is *economy*. Just as it is impossible to have an individual name for everything, so it is impossible to have an unlimited number of propositions to express the properties of an unlimited number of things. General names allow us to put in finite form knowledge about an infinite number of things.

Finally, note the important distinction between general and collective names; modern symbolic logic has essentially obliterated this distinction by calling them both sets. The importance of general names is that they are applicable to an indefinite (i.e., infinite) number of individuals. Hence propositions involving general names are true universally. This is not the case for collective names. I may make a true statement about the present members of the Navy which will be invalidated by the very next recruit. Hence propositions concerning collections (whether finite or infinite) do not have the universality of propositions concerning concepts. Therefore, in science and logic we are mostly concerned with general names and general propositions.

3. Concrete and Abstract

Summary: Mill makes a number of other distinctions in names, most of which are based on traditional scholastic logic. He defines a *concrete* name as a name which

stands for a thing, and an *abstract* name as a name which stands for an attribute of a thing. This distinction is orthogonal to that between general and singular names. We can have singular abstract names, such as 'visibleness' and 'squareness', which denote a single attribute. We can also have general abstract names, such as 'redness', which apply to a number of different shades of red. Note that 'whiteness' is an abstract name, the name of an attribute, while 'white' is a concrete name, the name of all white things.

4. Connotative and Non-Connotative

Summary: Mill next introduces what he claims is one of the most important distinctions: the difference between *connotative* and *non-connotative* names. "A non-connotative term is one which signifies a subject or an attribute only. A connotative term is one which denotes a subject and implies an attribute." Thus 'London' and 'whiteness' are non-connotative terms. 'White' is a connotative term because it denotes all white things, and implies or *connotes (con* = with, *notare* = to mark) the attribute whiteness, which all these things possess.

Consider the word 'man'. This term *denotes* Tom, Dick, Jane, and an indefinite number of other men, whether alive now or not. The term 'man' *connotes* a number of attributes, namely "corporeity, animal life, rationality, and a certain external form which, for distinction, we call the human." These attributes are essential to our calling a thing 'man'; anything which lacks even one of these attributes would not be called 'man'. But it is not always easy to decide the connotation of a term: "In some cases it is not easy to decide precisely how much a particular word does or does not connote; that is, we do not know (the case not having arisen) what degree of difference in the object would occasion of difference in the name."

An important problem for philosophers and scientists is to discover the proper connotation of a term, which is essentially the process of definition. Mill claims that when a term in common use is defined, the connotation should be chosen in such a way that

it alters as little as possible the denotation of the term and contradicts as few as possible of the propositions received as true about the things denoted.

Comments: It is not clear what, exactly, Mill means by connotation. He frequently says that the connotation of a term comprises those attributes whose presence causes us to apply the term to a thing. This might suggest that the connotation of a name is the same as its definition, but Mill later (Chapter 5) states, "In defining a name, however, it is not usual to specify its entire connotation, but so much only as is sufficient to mark out the objects usually denoted by it from all other known objects."

An alternate interpretation is that the connotation of a name is *all* of the attributes implied by the name. Thus the connotation of *man* would include rationality, animality, his distinctive shape, the ability to use language, an opposable thumb, and so on, for an unlimited number of attributes. This is not Mill' view, however, since in Chapter 7 he states, "Of all the innumerable properties known and unknown that are common to the class man, a portion only, and of course a very small portion, are connoted by its name; these few, however, will naturally have been thus distinguished from the rest either for their greater obviousness, or for their greater supposed importance." Also, he later distinguishes propositions that unfold the connotation of a term, such as 'Man is an animal', from those which express an "accidental" fact, such as 'Man is mortal'.

How can we tell whether or not a given attribute is connoted by a name? In Killick (1909) we find, "The best mode of determining whether a name connotes a given attribute is to ask, Whether, if that attribute were removed, the name would still be applied to the subjects? Does 'man' connote mortality? The test is, should we apply the name 'man' to beings exactly like men in other respects, but not mortal?" Although Killick does not answer the question in this particular case, presumably he would answer "Yes." On the other hand, if we ask whether we should apply the name 'man' to beings exactly like men in other respects, but not rational, presumably he would answer "No." I dare say that in common usage we would more likely apply the name 'man' to an irra-

tional being otherwise like man, than to an immortal being otherwise like man.

What, then, is the root of Mill's notion of connotation? It appears to be that the connotation "of a general term or concept A [is] made up of all those general terms or concepts B for which 'All As are B' is a necessary truth ...." ['extension and intension' in Flew (1979)]. The presumption is that manness *necessarily* implies rationality, but only *contingently* (accidently) implies mortality. Thus Mill's notion of connotation seems to hinge on the distinction between necessary and contingent truth, a distinction which, later, I will argue is fallacious[1]. For the time being the reader will be closer to the truth if he takes the connotation of a name to be all of the attributes that are truthfully predicable of everything denoted by the name. Thus, the connotation of *man* includes rationality, animality, an opposable thumb, the potential ability to cook food, the ability to fly to the moon, etc., etc. This is the interpretation of connotation that I will use throughout this work.

Mill's discussion of the connotation of proper names is also unsatisfactory. Joseph (1906) says about it, "He confounded different distinctions, and raised a controversy about the connotation of proper names, to which there has been no satisfactory issue, because he never clearly realized to himself what he meant by connotation ...." The basis of Mill's belief that proper names have no connotation is presumably the idea that proper names do not imply anything in a *necessary* way; everything we know about them is contingent. Thus the name 'Caesar' does not imply necessarily that Caesar is a man. But observe that if we do not know the name 'Caesar', then for us it neither connotes nor denotes. However, if we know the individual to whom this name refers then we know both the name's denotation and part of its connotation (in the wider sense of connotation suggested above). The same of course applies to general names. Hence, with this wider notion of connotation one can see that a proper name has, in fact, a very rich connotation, since an individual has "more" attributes than any general

1. The fallacy of the necessary/contingent dichotomy is discussed in Peikoff (1979), Quine (1953) and Hempel (1954).

class. (These issues are discussed at some length in Joseph (1906), Chapter 6.)

Mill identifies a very important point: the denotation and connotation are both essential parts of a (connotative) term. There is a tendency to first define a term by identifying certain attributes as the connotation of the term, and then to replace the term by its definition. This ignores the fact that a definition is *wrong* if it substantially alters the denotation of the term (or, at very least, it is the definition of some other term). Such "denotation shifting" is in fact a logical fallacy.

It can be argued that sometimes a definition must change the denotations of a term. For example, traditionally a whale might have been considered a fish because of its external form and its aquatic habitat. A better understanding of animal life later forced us to remove whales from the denotation of 'fish'. This is of course correct: we have altered the denotation of 'fish' on the basis of new evidence. More accurately, we have divided fish into true-fish and aquatic mammals. When we define these terms, we must preserve the denotation of each. If a definition were to substantially alter the denotation of a term, then a new term would be called for.

We can take Shannon's *Information Theory* as an example of this fallacy. It is widely recognized among scientists that Information Theory has little to do with information in the colloquial sense, and that Shannon's measure of information does not measure informativeness (e.g., a book of random numbers has the most information). To one who recognizes the limitations of formal Information Theory, this theory can be a powerful tool. Nevertheless, there have been many misapplications of Information Theory resulting from an identification of Shannon's *information* with the usual *information*. Indeed, books mistakingly making this identification are still common in the popular press.

The reader is also likely to encounter the terms *intension* and *extension*, which are often used as synonyms for *connotation* and *denotation*, although some writers make slight distinctions. We will adhere to Mill's terminology in this work.

5. Positive and Negative

Summary: Names can also be divided into *positive* and *negative*. Examples of positive names include *man*, *tree*, and *good;* examples of negative names include *not-man*, *not-tree*, and *not-good*. The distinction between positive and negative names is not a distinction of form (such as the possession of a negative prefix like *not-* or *un-)*, but a distinction of meaning. For example, the word *inconvenient*, though negative in form, is positive in meaning, since it expresses a positive attribute, the *presence* of some cause of discomfort or annoyance. Similarly, the word *innocent*, although positive in form, is negative in meaning, since it expresses a negative attribute, the *absence* of an illegal or unethical act.

Comments: The distinction between positive and negative names is crucial; unfortunately Mill's treatment of it is completely inadequate. In particular he barely treats the question of what makes a name positive or negative.

Since the distinction between positive and negative terms is one of meaning rather than form, it has largely been lost from modern formal logic. We will see later that this notion is crucial to the understanding of inductive proof.

In attempting to understand the difference between positive and negative names, we can begin with Aristotle, who said in *On Interpretation* (ii.16a30-33): "The expression 'not-man' is not a noun. There is indeed no recognized term by which we may denote such an expression, for it is not a sentence or a denial. Let it then be called an indefinite noun, since it refers to all kinds of things, non-existent as well as existent."

Whately (1868) also calls a negative term indefinite "in respect of its not defining and marking out an object, in contradistinction to this, a positive term is called Definite ... because it does thus define or mark out." In other words, definite terms mark out or limit our view to one particular class of things, or one thing, while indefinite terms exclude such a class or individual, leaving undetermined the individu-

als of which we speak.

Several logicians have noted that these purely negative terms have a limited value. De Morgan (1847) says, "There can be little effective meaning, and no use, in a classification which, because they are not men, includes in one word *not-man*, a planet and a pin, a rock and a featherbed, bodies and ideas, wishes and things wished for." Given the questionable value of negative terms, the reader might wonder why I dwell on them. The reason is that they are very common in modern symbolic logic, which considers *not-man* to be a predicate of essentially the same kind as *man*.

Joseph (1906) explains both the reason for negative terms and the fallacy in their use. "Such negative terms as these do not really figure in our thought; they are 'mere figments of logic'; Aristotle long ago pointed out that [not-man] was not properly a name at all ...." These negative terms result from an "attempt to reduce negative and affirmative judgements to a common affirmative type, by throwing the negative into the predicate ...." This "is not really defensible for the negative term [not-man] does not signify the nature of anything, and so is not really a term; it should, if it were a general term covering everything except the corresponding positive, be predicable of all subjects except [men] in the same sense; but there is no common character in all of these which it is intended to signify .... [I]t is clear that we have not resolved the negative into the affirmative form, when such affirmation can only be understood by restoration to the negative."

The indefinite character of negative terms has led logicians to distinguish between *contradictory* and *contrary* terms (see, for example, De Morgan (1847), Joseph (1906), Whately (1868), Jevons (1919), and Creighton (1906)). Contradictory terms such as *man* and *not-man* divide the universe into two mutually exclusive, exhaustive classes; everything is either *man* or *not-man*. Contrary terms divide some class within the universe into mutually exclusive, exhaustive subclasses. For example, *even-number* and *odd-number* are contrary terms; some things are neither even numbers nor odd

numbers, for example, Eucalyptus trees.

Contrary terms can also be characterized as positive or negative. Consider the terms *guilty* and *innocent;* these are contraries rather than contradictories, since there are things, such as numbers, that are neither guilty nor innocent. How can we distinguish positive and negative terms?

Positive terms are characterized by the presence of certain common attributes or qualities possessed by the things denoted by the term. Negative terms can only be characterized by the absence of such common attributes and qualities. For example, the positive term *man* is characterized by the conjunction of properties such as animality, rationality, and human form. Couldn't we similarly say that the negative term *not-man* is characterized by the disjunction of properties such as non-animality, non-rationality, and non-human form? Yes, but notice that we have characterized the positive term *man* in terms of other positive terms, and the negative term *not-man* in terms of other negative terms. We will always find it to be the case that positives can be defined by positives, but that a negative requires at least one negative in its definition.

This seems to result in an infinite regression; to avoid it we must know by some other means (other than definition in terms of positives or negatives) whether a term is positive or negative. In fact we can know this, since any term which is defined *ostensively* (by pointing) is by its nature positive. For example, to define the term *blue* we can point out a number of blue things. The hearer can then abstract out the common qualities of these things and understand what we mean by blue. It is not possible to communicate the term *not-blue* by pointing to things that aren't blue, because there are no common characteristics that the listener could abstract.

One can define a positive term by pointing to things that have some quality; one can't define a negative term by pointing to things that don't have a quality. Thus the distinction between positive and negative terms ultimately rests on the presence or

absence of definite sensory and perceptual qualities.

6. Relative and Absolute

Summary: The fifth leading division of names is into *relative* and *absolute* (i.e., *non-relative).* Examples of relative names are 'father', 'son', 'longer', 'shorter', and 'equal'. Their characteristic property is that they are always given in pairs (e.g. 'father', 'son'), although in some cases [reflexive relations] the two elements of the pair are the same (e.g., 'equal', 'equal'). "Every relative name which is predicated of an object, supposes another object (or objects), of which we may predicate either that same name or another relative name which is said to be the *correlative* [or *converse*] of the former."

The major reason for dwelling on relative names is that they provide insight into the nature of all attributes. "It is obvious, in fact, that if we take any two correlative names, *father* and *son* for instance, though the objects *denoted* by the names are different, they both, in a certain sense, connote the same thing. They don't connote the same attribute, but rather the same set of facts which we mean when we say A is the father of B and B is the son of A." "In this manner any fact, or series of facts, in which two different objects are implicated, and which is therefore predicable of both of them, may be either considered as constituting an attribute of the one, or an attribute of the other. This set of facts is what the schoolmen called the *fundamentum relationis*, or foundation of the relation."

3. Of the Things Denoted by Names

1. Necessity of an enumeration of Nameable Things

Summary: Mill has discussed proofs, the constituents of proofs, which are propositions, and the constituents of propositions, which are names - "If, therefore, we knew what all names signify, we should know everything which, in the existing state of human knowledge, is capable either of being made a subject of affirmation or denial or of being itself either affirmed or denied of a subject." Thus we will attempt "an enumeration of all kinds of things which are capable of being made predicates or of having any thing predicated of them ...."

Aristotle was the first to attempt an enumeration of "all things capable of being named; an enumeration by the *summa genera*, i.e., the most extensive classes into which things could be distributed ...." These highest predicates were called the *categories*. Table 1 is Aristotle's list of categories.

TABLE 1. Aristotle's Categories

Substance	[e.g., man or horse]
Quantity	[e.g., one foot long]
Quality	[e.g., blue]
Relation	[e.g., double]
Action	[e.g., to cut]
Passion	[e.g., to be cut]
Place	[e.g., in the market-place]
Time	[e.g., yesterday]
Position	[e.g., sitting]
State	[e.g., armed]

Mill says that this list of categories is unphilosophical, superficial, redundant and defective. That is, many of the distinctions are merely verbal, several of the categories overlap, and some things (such as states of consciousness) do not fall under any of the categories.

Comments: In Aristotle's defense it should be noted that he acknowledged that the categories overlap, and never claimed that the list was exhaustive. Also, Greek philosophy, like many modern philosophies, often confused linguistic distinctions with logical

distinctions. Finally, as I will discuss later, the notion of a *summum genus* is itself fallacious.

2. Feelings, or states of Consciousness

Summary: Mill uses *feeling* in a philosophical sense, i.e., to mean any state of consciousness. "Feeling, in the proper sense of the term, is a genus, of which sensation, emotion, and thought, are subordinate species." Mill cautions us to carefully distinguish objects from our ideas of them. "Even imaginary objects (which are said to exist only in our ideas) are to be distinguished from our ideas of them."

Similarly, sensations (which are mental experiences) are distinguished from the objects which produce them and from the attributes of these objects which cause them to excite these sensations. Thus, the sensation of white must be carefully distinguished from the objects which produce this sensation, which we call *white*, and from the quality, which we call *whiteness*, that causes these objects to produce this sensation.

3. Feelings must be distinguished from their physical antecedents

Summary: Another distinction that must be carefully maintained is that "between the sensation itself and the state of the bodily organs which precedes the sensation and which constitutes the physical agency by which it is produced."

Comments: These distinctions - between sensations and the objects that product them, and between sensations and their physical antecedents - are often ignored by contemporary philosophers. The important point is that sensations are *primary;* it is these that we have a direct awareness of, not objects or nerve impulses. Before we can be aware that there even *are* nerves, we must be able to see, so that we can look through a microscope or watch an oscilloscope screen.

To reiterate, *epistemologically* sensations are primary: experience of them is prior to our experience of objects or nerve impulses. *Causally* (or ontologically) objects and nerve impulses may be prior to sensations, but that is a question for science to decide.

To accomplish this, scientists will make use of the sensations they experience.

4. Perceptions

Summary: Mill comments on the notion of a ***perception*** as an intermediate link between the stimulation of our sense organs and the resulting sensation in our minds. He says that a perception "consists in the recognition of an external object as the exciting cause of the sensation." This notion is dismissed by Mill: "When a stone lies before me, I am conscious of certain sensations which I receive from it; but if I say that these sensations come to me from an external object which I *perceive*, the meaning of these words is that, receiving the sensations, I intuitively ***believe*** that an external cause of those sensations exists." Mill then says that the "laws of intuition and the conditions under which it is legitimate" fall within the field of psychology rather than logic.

Comments: The notion of a ***perception*** is, in fact, central to logic, for it is perceptions, not sensations, that form the raw data of observations. Thus, contrary to the positivists, we do not see patches of isolated colors. Rather, we see organized groups of sensations that are automatically integrated by our visual mechanism. It is by a process of abstraction that we can come to think of sensations (such as the color red) in isolation from the perceptions incorporating them. The above statements are validated by experiments that each reader must perform individually in the laboratory of his own mind. That perceptions are primaries renders invalid any alternate attempt at their validation (say, by studying nerve impulses).

In the previous paragraph, we have used ***perception*** to refer to an automatically integrated system of sensations. There is no reference in this definition to the existence of external causes. Mill is correct when he says that a perception does not give us immediate knowledge of an external object — we are all familiar with the phenomenon of hallucinations. In a wider sense, however, every perception is the result of an external condition: the relationship between our perceptual system (comprising, so far as we know, the brain and the sense organs) and the rest of the

external world. It is one of the tasks of science to sort out those characteristics of perception that are caused by our perceptual system (which is part of the external world) from those characteristics which are caused by the objects of our observation.

Our perceptual system is the instrument through which we observe reality. Like any instrument, it has a definite identity and, hence, definite limitations. We do not consider measurement impossible because our instruments have limitations. Likewise, the impossibility of knowledge is not implied by the existence of limitations in our perceptual system.

In summary, every perception has a cause in the external world. That cause is a complex of the observer and the observed. We will see later that a major task of science is to attribute characteristics of a perception to one or the other of these two external objects.

5. Volitions and Actions

Summary: Mill observes: "When we speak of sentient beings by relative names, a large portion of the connotation of the name usually consists of the actions of those beings; actions past, present, and possible or probable future." What is an action? It is "not one thing, but a series of two things: the state of mind called a volition, followed by an effect." For example, when I form the volition to move my arm, it moves, unless it is paralyzed or restrained.

Comments: Mill alludes to, but does not identify, the fact that a volition is an irreducible primary, just like a perception. That is, we do not see any "substeps" in an act of perception. That is, we do not see any "substeps" in an act of perception; it is literally *immediate* (no mediate steps). This *epistemological* primacy of perceptions does not contradict any *causal* primacy that might be identified by studying physiology.

Similarly, a volition is immediate; I observe no steps between my volition to move my arm and the motion of my arm. There is of course nothing mystical or superna-

tural about the irreducibility of a volition; it is an epistemological irreducibility, not a causal (or ontological) irreducibility. Epistemological irreducibility does not contradict any explanation by science of volition in terms of nerve impulses or other physical phenomena.

Thus, we can call a perception an *afferent primary* and a volition an *efferent primary*.

6. Substance and Attribute

Summary: Having dealt with feelings (sensations, thoughts, emotions, and volitions), Mill proceeds to the two remaining classes of namable things, substances and attributes. Previous logicians, Mill observes, have usually drawn this distinction on the basis of a word's grammatical function (e.g., noun or adjective) rather than on the basis of distinctions among the things the words name. In answer to the classical question, whether substances can exist without attributes, or attributes without substance, Mill says, "we can no more imagine a substance without attributes than we can imagine attributes without a substance." "Whiteness, without any white thing, is a contradiction in terms."

7. Body

Summary: Mill defines a *body* as "the external cause to which we ascribe our sensations." Are we justified in ascribing our sensations to an external cause? Certainly "a part of our notion of a body consists of the notion of a number of sensations of our own, or of other sentient beings, habitually occurring simultaneously." Is there any reason to presume a *substratum* underlying these recurring groups of sensations? After reviewing the theories of Locke, Hartley, Hamilton, and Berkeley, Mill concludes that "of the outward world, we know and can know absolutely nothing except the sensations which we experience from it."

Comments: There are many hypotheses that can explain our sensations: external

reality, hallucinations, perceptual errors, etc. In most cases the *simplest* hypothesis is that external reality is the cause of our sensations. Why we choose the simplest hypothesis, and how we judge simplicity, are topics we take up in Book III, *Of Induction*.

8. Mind

Summary: Mill says that, just "as our conception of a body is that of an unknown exciting cause of sensations, so our conception of a mind is that of an unknown recipient or percipient of them; and not of them alone, but of all our other feelings." "As bodies manifest themselves to me only through the sensations of which I regard them as the causes, so the thinking principle, or mind, in my own nature makes itself known to me only by the feelings of which it is conscious."

Comments: Recall (Section 3) that perceptions, not raw sense data, are primary, and that perceptions are automatically integrated sensations (both external and internal). It seems that our notion of self is a perception automatically integrated from internal sensations of our own mental processes. Thus, our notion of 'self' is an epistemological primary. (See Nozick (1981), Chapter 1, for a good discussion of borderline cases in the notion of 'self'.)

9. Qualities

Summary: Mill turns from substances to *attributes*, which he says are of three kinds; *qualities*, *quantities* and *relations*. He says that "if we know not and cannot know anything of bodies but the sensations which they excite in us or in others, those sensations must be all that we can, at bottom, mean by their attributes ...."

Mill asks what it is that we mean when we ascribe a quality, such as whiteness, to some object, such as snow. Do we mean only that when snow is presented our sense organs a certain sensation (that we call white) is experienced? Or do we mean that the object possess some inherent "potency," the attribute whiteness, which produces in us

that sensation? In short, is there any difference between a quality and a sensation?

Although Mill says that this distinction is not important to logic, he does claim that, "when we say that snow is white because it has the quality of whiteness, we are only re-asserting in more technical language the fact that it excites in us the sensation of white. If it be said that the sensation must have some cause, I answer, its cause is the presence of the assemblage of phenomena which is termed the object." Mill says that there is no point in interpolating a "potency" between the object and the sensation.

10. Relations

Summary: "The *qualities* of a body, we have said, are the attributes grounded on the sensations which the presence of that particular body to our organs excites in our minds. But when we ascribe to any object the kind of attribute called a Relation, the foundation of the attribute must be something in which other objects are concerned besides itself and the percipient." Two things can be said to be related when "there exists or occurs, or has existed or occurred, or may be expected to exist or occur, some fact or phenomenon, into which the two things ... both enter as parties concerned. This fact, or phenomenon, is what the Aristotelian logicians called the *fundamentum relationis.*"

Relations can be based on complicated series of facts (as are legal relations), or can be based on very simple facts. An example of the latter is our experience of two events as either simultaneous or successive. Mill claims that these latter relations are perceptual primaries that canno be analyzed further.

11. Resemblance

Summary: Mill claims that two other sorts of relations, likeness and unlikeness, are also primaries. "Resemblance is evidently a feeling, or state of the consciousness of the observer." These relations are not capable of [epistemological] analysis because they are presupposed in every analysis. "Likeness and unlikeness, therefore, as well as

antecedence, sequence, and simultaneousness, must stand apart among relations, as things *sui generis.*" Certainly, however, complex cases of likeness and unlikeness *can* be resolved into simpler ones. "All likeness or unlikeness of which we have any cognizance resolve themselves into likeness and unlikeness between states of our own, or some other, mind."

Comments: A fundamental requirement of good science is to recognize epistemological primaries. To attempt to analyze a primary is an error, specifically, a *category error.* This is because it is an error to attempt to base an epistemological primary upon something that is not a primary.

12. Quantity

Summary: Mill asks us to consider two comparisons: the first, between a gallon of water and ten gallons of water; the second, between a gallon of water and a gallon of wine. In the first case we say that they different in *quantity*, in the second in *quality.* Both of these assertions are grounded on differences in the sensations they excite. But what is the distinction between a *quantitative* difference and a *qualitative* difference? Mill says, "This likeness and unlikeness I do not pretend to explain, no more than any other kind of likeness or unlikeness."

Comments: Certainly, at base, our notion of quantity is an epistemological primary, just like our notions of color and shape. Depending on circumstances (such as arrangement) we can directly perceive numbers up to about a dozen. That is, our perceptual system tells us automatically whether some number of objects is equal, greater, or less than some other number of objects. This automatic perceptual integration is the basis for our recognition of quantity, as it is for our recognition of color, shape, etc.

Of course, we extend our notion of quantity beyond those quantities that are immediately perceivable, just as we extend our notions of color and shape beyond the immediately perceivable. (For example, certain colors and shapes are indistinguish-

able on a direct perceptual basis.) This extension of ideas from the directly perceivable primaries to non-primaries is the basis for *measurement*. But, to understand this extension process, we must investigate more carefully what distinguishes quantity from other attributes, which we will do in the comments accompanying Book II, Chapter 6 (On the Science of Number).

13. Attributes Concluded

Summary: "Thus, then, all the attributes of bodies which are classed under quality or quantity are grounded on the sensations which we receive from those bodies ...." The same applies to every attribute of mind, which "consists either in being itself affected in a certain way or affecting other minds in a certain way." This provides the key to the analysis of attributes such as 'beauty': "As we thus ascribe attributes to minds on the ground of ideas and emotions, so may we to bodies on similar grounds, and not solely on the ground of sensations: As in speaking of the beauty of a statue, since this attribute is grounded on the peculiar feeling of pleasure which the statue produces in our minds, which is not a sensation, but an emotion." Thus, all attributes are ultimately grounded on feelings, i.e., states of consciousness.

14. Recapitulation

Summary: In summary, the things which can be named are in three categories. First, there are feelings (states of consciousness), which are of four sorts: sensations, thoughts, emotions, and volitions. Second, there are substances, which are of two sorts: bodies and minds. Finally there are attributes, which are of three sorts: qualities, relations, and quantities. But, Mill has argued that all attributes are reducible to sensations or states of consciousness. These considerations lead Mill to the following enumeration of the categories of all namable things:

1. *Feelings*, or states of consciousness.

2. *Minds*, which experience those feelings.

3. *Bodies*, or external objects which excite certain of those feelings.

4. *Successions and coexistences, likeness and unlikeness*, between feelings or states of consciousness.

Mill concludes this section by distinguishing *psychological* or *subjective* facts, which are composed solely of feelings or states of consciousness, from *objective* facts, which also incorporate substances and attributes. "We may say, then, that every objective fact is grounded on a corresponding subjective one, and has no meaning to us (apart from the subjective fact which corresponds to it), except as a name for the unknown and inscrutable process by which that subjective or psychological fact is brought to pass."

Comments: This latter view, that objective facts are based on subjective facts, is essentially the view of British empiricism, in the tradition of Locke, Berkeley, and Hume. The intended readers of this work, scientists, generally don't need to be convinced that the real world exists. Nevertheless, it may be worthwhile to spend a few sentences to discuss Mill's subjective empiricism. Certainly, if one doubts the existence of the real world, then there is very little reason to engage in science, since the purpose of science is to give us knowledge *about* the real world. That is, the existence of the real world is a presupposition of the study of logic or the scientific method. To put this another way, the purpose of science is to establish true propositions, i.e., statements that correspond with the state of affairs in the real world (the so-called correspondence theory of truth). Without the real world, there is no significance to *truth*. Hence, we will assume that any reader interested in Mill's scientific method does not question the existence of reality.

Although it is certain *that* reality is, it is not certain *what* it is. The establishment of the *nature* of reality (as opposed to its *existence)* is, of course, the purpose of science. However, the only way we know reality is through our senses (including our introspective awareness of our own mental states). This is the sense in which Mill is correct:

epistemologically, that is, in the order in which we gain knowledge, subjective facts are prior to objective facts. Indeed, this is the essence of objectivity: we only believe wh- we ultimately can see, hear, touch, etc., either directly or indirectly. The goal of scientific methodology is the elaboration and refinement of the previous sentence.

4. Of Propositions

1. Nature and office of the copula

Summary: A *proposition* is a sentence in which a predicate is affirmed or denied of a subject. To accomplish this affirmation or denial, a *copula* is necessary (usually the verb *to be).* Hence, the first division of propositions is into *affirmative* and *negative*. An affirmative proposition is one in which the predicate is *affirmed* of the subject; a negative proposition is one in which the predicate is *denied* of the subject.

2. Affirmative and Negative Propositions

Summary: The distinction between affirmative and negative propositions is real. Some writers, such as Hobbes, have claimed that negative propositions are just disguised affirmative propositions. For example, 'Caesar is not alive' is really a disguised form of 'Caesar is non-alive'. But this analysis has just replaced one proposition denying a positive predicate with another affirming a negative predicate. This has accomplished nothing. "The distinction between affirming and denying is real and is not to be got rid of by a verbal juggle." Thus, "when we affirm a negative name, we really affirm the *absence*, not the *presence*, of anything; not that something *is*, but that it is *not*."

Comments: The distinction between affirmative and negative propositions is an essential one that has been abandoned in symbolic logic. The value of this distinction will be more apparent when I discuss induction and scientific method in Book III. As Mill notes, the distinction between affirmative propositions is grounded on the distinction between positive and negative terms, which I discussed in Chapter 2.

3. Simple and Complex

Summary: "A simple proposition is that in which one predicate is affirmed or denied of one subject. A [complex] proposition is that in which there is more than one predicate, or more than one subject, or both." Complex propositions in turn can be divided into

categorical and *hypothetical* propositions.[2]

Categorical propositions are those "in which the assertion is not dependent on a condition." Categorical complex propositions are equivalent in meaning to two or more simple propositions. For example, 'Caesar is dead, but Brutus is alive' is equivalent to:

1. Caesar is dead.

2. Brutus is alive.

3. Propositions 1 and 2 should be thought of together.

4. There is a contrast between propositions 1 and 2.

The function of the particle 'but' is to abbreviate propositions 3 and 4.

Unlike the categorical complex propositions just considered, a hypothetical proposition is not "a mere aggregation of simple propositions." Although a hypothetical proposition may contain several subjects and several predicates, it makes only one assertion. Take as an example one kind of hypothetical proposition, the *conditional* proposition. The conditional proposition 'If A is B, C is D' is just an abbreviation for 'The proposition C is D, is a legitimate inference from the proposition A is B'. The latter is a simple proposition whose subject is 'the proposition C is D' and whose predicate is 'a legitimate inference from A is B'. Thus, a conditional proposition is a simple proposition whose terms are the names of propositions.

"Like other things, a proposition has attributes which may be predicated of it." For example, 'That the whole is greater than its parts, is an axiom in mathematics' is a proposition whose subject is a proposition.

Another common kind of hypothetical proposition is the *disjunctive*, for example, either A is B or C is D. Whately (1868) and others have shown that this is resolvable into the two conditionals:

---

2. All simple propositions are categorical.

- If A is not B, C is D.
- If C is not D, A is B

Comments: Mill's and Whately's explication of the inclusive *or* connective is easy to see in the propositional calculus:

$(\sim P \rightarrow Q) \ \& \ (\sim Q \rightarrow P)$

$(\sim\sim P \vee Q) \ \& \ (\sim\sim Q \vee P)$

$(P \vee Q) \ \& \ (P \vee Q)$

$P \vee Q$

On the other hand, Mill's definition of the conditional proposition cannot be expressed in symbolic logic because it mixes the object language and metalanguage levels. That is, it is a proposition about the inferability of propositions. To express such a notion, we have to use techniques like Godel's for embedding metalanguage propositions in the object language. Traditional logic avoids the object language/metalanguage distinction.

4. Universal, Particular, and Singular

Summary: Propositions may be divided into three major classes on the basis of the degree of generality which the subject is understood to have. These classes are: [3]

- *Universal*, for example, *All men* are mortal.
- *Particular*, for example *Some men* are mortal.
- *Singular*, for example, *Julius Caesar* is mortal.

A proposition is *singular* when its subject is an individual name. "When the name which is the subject of the proposition is a general name, we may intend to affirm or deny the predicate, either of all the things the subject denotes, or only of some." In the former case the proposition is universal, in the latter particular. Since in a singular

3. Following Mill, we have used the traditional terms, although Bain's *(Logic)* terms *total* and *partial* are perhaps more descriptive.

proposition the predicate is affirmed or denied of the entire subject, a singular proposition is usually considered a universal proposition.

Traditionally, an occurrence of a name is said to be *distributed* when it stands for each and every individual the name denotes and is said to be *undistributed* otherwise. Thus, in a universal proposition the subject is distributed, but in a particular proposition the subject is undistributed. Clearly, the subject is distributed in a singular proposition.

Comments: It is worth adding that in a negative proposition the predicate is distributed and in an affirmative proposition the predicate is undistributed. Thus we have the classification shown in Table 2.

TABLE 2. Classification of Propositions

Quantity	Quality	Distribution		Example
		Subject	Predicate	
universal	affirmative	D	U	All S and P
particular	affirmative	U	U	Some S are P
universal	negative	D	D	No S are P
particular	negative	U	D	Some S are not P

TABLE 3. Set Expressions Equivalent to Types of Propositions

Quantity	Quality	Distribution	Set Expression
universal	affirmative	DU	$S \subseteq P$
particular	affirmative	UU	$S \between P$
universal	negative	DD	$S \subseteq \sim P$
particular	negative	UD	$S \between \sim P$

Since the extension of a general term is a class, we can express these propositional forms in the algebra of classes. This is shown in Table 2. Here we have used '$S \between P$' as an abbreviation for $S \cap P \neq \phi$. Notice that the forms containing '$\subseteq$' distribute the subject and those containing '~' distribute the predicate. Furthermore, note that if we always write a negative term with a '~' sign and a positive term without one, then the form of the proposition will be obvious regardless of "verbal juggling." Thus, whether we say 'some men aren't mortal' or 'some men are non-mortal', the form is 'men $\between$ ~mortal'. The symbolic notation also simplifies transforming propositions into

equivalent forms. For example, since $S \subseteq \sim P$ is equivalent to $P \subseteq \sim S$ (as we can convince ourselves with Venn Diagrams), we know that 'no man is a fish' is equivalent to 'No fish is a man'. Of course, the early development of the algebra of classes by De Morgan and Boole took place after Mill's *Logic* had been written, so he didn't have the benefit of notations such as this.

5. Of the Import of Propositions

1. Is Proposition a Relation Between Two Ideas?

Summary: Mill asks, "What is that which is expressed by the form of discourse called a proposition, and the conformity of which to fact constitutes the truth of the proposition?" Since a proposition consists of two terms (a subject and a predicate) connected by a copula, we must first understand what the terms represent, and then ask what kind of connection between them the copula asserts.

Philosophers, "from Descartes downward, and especially from the era of Leibnitz and Locke," have taken a proposition "to consist in affirming one *idea* of another." will disagrees with this view. "When I say that fire causes heat, do I mean that my idea of fire causes my idea of heat? No, I mean that the natural phenomenon, fire, causes the natural phenomenon, heat." In fact he claims that focusing on ideas of things rather than the things themselves was the cause of much of the sterility of pre-scientific reasoning. The scientific view is that propositions "are not assertions respecting our ideas of things, but assertions respecting the things themselves."

2. Is a Proposition a Relation Between the Meanings of Two Names?

Summary: Mill next addresses Hobbes' view that a proposition signifies "the belief of the speaker that the predicate is a name of the same thing of which the subject is a name." When we assert that all oxen ruminate it is certainly true that all the individuals denoted by the name 'ox' are asserted to be among all the individuals denoted by the name 'ruminating'. Thus Hobbes' analysis is true of all propositions. And in fact it is the entire meaning of some propositions, which "only shows what an extremely minute fragment of meaning it is quite possible to include within the logical formula of a proposition."

Mill says that this account of meaning could be considered adequate only because Hobbes, in common with the nominalists, sought the meaning of terms entirely in their

denotation and ignored their *connotation*. What is wrong with this view? As explained earlier, "the meaning of all names, except proper names and that portion of the class of abstract names which are not connotative, resides in the connotation." Otherwise we could not explain the meaning of a proposition such as 'diamonds are combustible'. Certainly when mankind fixed the meaning of the word 'combustible' they did not know that the individuals denoted by 'diamond'. The names *happen* to fit the same objects because of a certain *fact*, which fact was not known when the names were invented. Thus "the objects are brought under the name by possessing the attributes connoted by it: but their possession of the attributes is the real condition on which the truth of the proposition depends; not their being called by the same name. Connotative names do not precede, but follow, the attributes which they connote."

3. Is a Proposition an Expression of Class Membership?

Summary: "The most generally received notion of a predication decidedly is that it consists in referring something to a class, that is, either placing an individual under a class, or placing one class under another class. ... If the proposition is negative, then ... it is said to exclude something from a class." This is essentially the same as Hobbes' theory, since "a class *is* absolutely nothing but an indefinite number of individuals denoted by a general name."

Mill believes that this theory is an example of a common logical error, *hysteron proteron*[4] (last first), or "explaining a thing by something which presupposes it." This is because, only after having judged that snow is white and several other objects are white do I gradually begin to think of white objects as forming a class. "We place the individual in the class because the proposition is true; the proposition is not true because the object is placed in the class." Mill claims that this view seems to treat classes as preexisting, as though having once been laid down by the framers of language. Furthermore, we must think of the meaning as constantly changing as individuals become or

4. ὑστερον προτερον.

cease to be members of the class. Thus, a name would have no definite meaning. "The only mode in which any general name has a definite meaning is by being a name of an indefinite variety of things, namely, of all things, known or unknown, past present, or future, which possess certain definite attributes." Thus, attributes are prior to classes.

Mill concludes by noting the dominance of these two erroneous views: "Since the revolution which dislodged Aristotle from the schools, logicians may almost be divided into those who have looked upon reasoning as essentially an affair of ideas and those who have looked upon it as essentially an affair of names."

Comments: This trend has certainly continued through the 20th century, although the idea-oriented viewpoint has tended to be displaced by the class-oriented view begun by Boole, Russell and Whitehead, and the formal view that descended from symbolic logic. Although these views capture most of deductive reasoning, they are inadequate for explaining induction. The following sections present Mill's attempt to solve these problems.

4. What it Really Is

Summary: Consider a singular proposition, such as 'Socrates is wise'. The meaning of a proposition such as this is that 'the individual thing denoted by the subject has the attributes connoted by the predicate'. Next, consider a proposition whose subject is connotative, such as 'All men are mortal'. In this case we are also asserting that the objects denoted by the subject possess the attributes connoted by the predicate. However, in this case the objects are not individually pointed out--they are identified by some of their attributes, namely those connoted by the name 'man'. Thus, a proposition of this form means that "whatever has the attributes connoted by the predicate; that the latter set of attributes *constantly accompany* the former set."

Mill carries this analysis one step further by recalling that "every attribute is *grounded* on some fact or phenomenon, either of outward sense or of inward conscious-

ness, and that to *possess* an attribute is another phrase for being the cause of, or forming a part of, the fact or phenomenon upon which the attribute is grounded. ... The proposition which asserts that one attribute always accompanies another attribute really asserts ... that one phenomenon always accompanies another phenomenon ...."

Comments: There are two ways to interpret Mill's explanation, based on the exact notion of connotation we adopt. If by the connotation of a term we mean *all* the attributes shared by the individuals denoted by that term, then his analysis is correct. For example, among the many attributes connoted by *man* we find the attributes connoted by *mortal*. This is what we mean when we say 'all men are mortal'. It is of course one of the tasks of science to determine whether the attributes connoted by *mortal* are among those *connoted* by man.

Another interpretation is that the connotation of a term includes only certain *essential* attributes. For example, the connotation of *man* might by rational animal, or featherless biped. In this case the connotation of *man* does not include the connotation of *mortal*. By this view 'man' can be considered to be just an abbreviation for 'rational animal' or 'featherless biped'. This analysis can easily be seen to be inadequate, since we can easily imagine discovering featherless bipeds that we would not be willing to call men. The The reason is that they would not share the other attributes of men, such as rationality. Thus the connotation must include all the attributes.

We can see in the above interpretation a different class-oriented analysis of propositions. 'All men are mortal' does not mean that the denotation of *man* is a subclass of the denotation of *mortal*, but that the connotation of *man* is a superclass of the connotation of *mortal*.

5. What it Is that Propositions Assert or Deny

Summary: "The object of belief in a proposition, when it asserts anything more than the meaning of words, is generally ... either the co-existence or the sequence of two

phenomena." For example, when we say, "A generous person is worthy of honor," we "affirm that wherever and whenever the inward feelings and outward facts implied in the word generosity have place, then and there the existence and manifestation of an inward feeling, honor, would be followed in or minds by another inward feeling, approval."

Although propositions asserting sequences and co-existences among phenomena are the most common, "we make propositions also respecting those hidden causes of phenomena, which are named substances and attributes." As noted previously, though, "no assertion can be made, at least with a meaning, concerning these unknown and knowable entities, except in virtue of the phenomena by which alone they manifest themselves to or faculties." Thus, propositions asserting the co-existence or the sequence of substances and attributes reduce to propositions concerning the co-existence and sequence of phenomena. [This view is known as *phenomenalism*].

Besides propositions that assert co-existence and sequence, there are those that assert simple existence and causation. Some logicians, such as Bain, have asserted that the concept of simple existence is empty. His Law of Relativity says that things can be perceived or apprehended only by contrasting them with other things. But, since "we have no other class to oppose to Being, or fact to contrast with Existence," these words are merely "fictitious and unmeaning language." Mill disagrees. The meanings of *existence* and *being* lie in the fact that to exist is to excite, or be capable of exciting, *any* sensations or states of consciousness ...." A thing can't *be* without being *something*. Causation and existence are discussed further in Book III.

"To these four kinds of matter-of-fact or assertion must be added a fifth, resemblance. This was a species of attribute which we found it impossible to analyze; for which no *fundamentum* distinct from the objects themselves could be assigned." Thus, a statement, such as "This color is like that color," cannot be analyzed [epistemologically] into any more basic propositions.

"It is sometimes said that all propositions whatever of which the predicate is a general name do, in point of fact, affirm or deny resemblance." There is only a slight degree of foundation for this remark, for although the arrangement of things into classes in based on resemblance, it is not a mere general resemblance, but rather a resemblance that "consists in the possession by all those things [in the class] of certain common peculiarities." It is those peculiarities "which the terms connote, and which the propositions consequently assert, not the resemblance."

There are some exceptional classes that are founded on general unanalyzable resemblance. "The classes in question are those into which our simple sensations, or other simple feelings, are divided." Thus when I classify things as white, the basis for this classification is an unanalyzable [i.e., epistemologically primary] sensation of resemblance.

"Existence, co-existence, sequence, causation, resemblance: one or other of these is asserted (or denied) in every proposition which is not merely verbal. This five-fold division is a exhaustive classification of matters-of-fact, of all things that can be believed or tendered for belief, of all questions that can be propounded, and all answers that can be returned to them."

Comments: The position of *phenomenalism*, that all proposition are ultimately propositions about sensory phenomena, has developed into one of the dominant modern schools of the philosophy of science, the *logical positivism* of the Vienna Circle originated by Mach, Schlick and others. Against this position we can use Mill's own arguments (Section 1). When we say that an eclipse is caused by the moon coming between us and the sun, we clearly intend to say something about these objects in reality (i.e., the sun, moon and ourselves), not our sensations of these things. We use different verbal forms to talk about sensations. For example, "Our sensation of the sun is caused by its light falling on our retina, which causes impulses to travel down the optic nerve ...," and so forth. Mill would be more correct in saying that propositions concerning

substances and attributes are in fact intended as statements about the real world, but that the *evidence* for these propositions are propositions about the phenomena we experience directly.

6. Propositions with Abstract Terms

Summary: In the previous analysis Mill has addressed only propositions whose terms are concrete. However, since the meaning of such a proposition is based on the attributes which its terms *connote*, it is easy to extend the analysis to propositions whose terms are abstract, since abstract terms directly *denote* attributes. For example, the proposition, 'Prudence is a virtue', in which the terms are abstract, may be rendered 'All prudent persons, *in so far as* prudent, are virtuous', in which the terms are concrete.

In the previous section Mill showed that whenever a proposition has a concrete predicate, what we are predicating is an existence, co-existence, or resemblance. The interconvertibility of abstract and concrete terms leads Mill to conclude that an attribute is necessarily either an existence, co-existence, causation, sequence or resemblance."

Comments: Although in common discourse concrete propositions are the more common, the above analysis suggests that semantically abstract propositions are more fundamental. This is because abstract terms directly denote the characteristic attributes, whereas concrete terms only connote them, and the meaning of a proposition is based on these attributes.

6. Of Propositions Merely Verbal

1. Essential and Accidental Propositions

Summary: Mill has already refuted the Conceptualists' claim, that propositions state relations between ideas, and the Nominalists' claim, that propositions express the agreement or disagreement between the meanings of names. He has claimed that propositions "assert five different kinds of matters of fact, namely, Existence, Order in Place, Order in Time, Causation, and Resemblance. ..."

Mill reminds us, however, that there is a class of propositions that "do not relate to any matter of fact ... at all, but to the meaning of names." These would not be worth spending much time on, but for the fact that they occupy "a conspicuous place in philosophy" and that some philosophers regard them as expressing the most essential truths.

Comments: There are two senses in which we could say that proposition is about the meaning of names. One expresses a matter of fact about language, namely, that a given configuration of sounds or letters refers to a given concept. For example, when I say '*ἀνθρωπος* is the Greek word for *man*', I assert a matter of fact about the Greek language. However, when I assert 'man is the rational animal', I am not stating a matter of fact about English. Rather, I am stating a fact of fundamental importance about the entities which we group together under the concept *man*. Such a definition is more than simply an assertion about the way English speakers use the word 'man'.

2. Essential Propositions are Identical Propositions

Summary: The Schoolmen divided the attributes as anything into two classes: the *essential* attributes and the *accidental* attributes. The essential attributes were considered to be part of the *essence* of a thing, and thus go deeper than the *accidents*. For example, rationality was considered part of the essence of man, whereas that he cooks his food was considered an accident (i.e., not *essential).* This theory was based

on the Schoolmen's view that an object borrowed some of its properties form a universal substance (an essence) and that the rest belonged to it individually (were accidental). Although this view is often attributed to Aristotle, Mill notes that in the *Categories* Aristotle expressly denies that general properties inhere in a subject; they are merely predicated of it.

Following Locke, Mill claims that an essential proposition only unfolds the meaning of a term. For example, the word ***man*** connotes all of man's attributes, including corporeity, rationality, and being living. Thus, when we say 'Man is rational' we have merely singled out one of the attribute connoted by *man*. Thus, since the essences of classes are merely the signification of their names, and the signification of a name is just its connotation, we can see that all propositions that have been called essential are in fact identical.

Is there any value, then, to an essential proposition? Since it states part of the meaning of a term, it can only be informative to someone who does not already know the full meaning of the term. Thus, Mill says that the only really useful kinds of essential propositions are definitions. Even here he notes, "In defining a name, however, it is not usual to specify its entire connotation, but so much only as is sufficient to mark out the objects usually denoted by it from all other known objects. And sometimes, a merely accidental property, not involved in the meaning of the name, answers this purpose equally well."

Comments: What, then, can we make of this distinction between essence and accident, between verbal and real propositions? There certainly seems to be a distinction between a proposition such as 'Man is rational' and propositions such as 'Man can cook his food,' 'Man has five fingers,' and 'Man domesticates animals.' What is this difference? They all state properties of men, but some seem more *fundamental*, (more essential), while others seem less fundamental. What makes some properties seem more fundamental?

As noted before, one aspect of this distinction is *explanatory power*. The rationality and animality of man implies that he can cook food, provide shelter for himself, compose symphonies, design computers, etc. Thus, we feel we have got to man's essence, when we realize he is a rational animal. Notice, however, that in this sense it is no accident that man builds houses; it is a consequence of his rationality and animality.

Some properties of man do seem to be accidents: that man has five fingers on each hand does not seem to be a consequence of him being a rational animal. Further, this property doesn't seem to explain many others: it gives us no hint why he cooks or designs computers. Yet having five fingers is no less a property of man than is having a rational faculty. In this sense there is nothing accidental about it.

Thus, if we take connotation in the wide sense, to mean all the properties, known and unknown, shared by the members of the denotation of a term, then we can see that any true, universal affirmative proposition only "unfolds" what is already contained in the connotation of the subject. Yet we can hardly call such propositions "merely verbal."

Should we then discard the notion of essential properties? No, for it expresses a useful epistemological fact: that at given point in our understanding of a subject some propositions have greater explanatory value than others. Peikoff (1979) says, "To designate a certain characteristic as 'essential' or 'defining' is to *select*, from the total content of the concept, the characteristic that best condenses and differentiates that content in a specific cognitive context." Note that properties are not *inherently* essential: whether they are essential or not depends on their explanatory value relative to our knowledge. "The characteristic(s) which most fundamentally distinguishes a certain type of entity from all other existents known at that time, may not do so within a wider field of knowledge, when more existents become known and/or more of the entity's characteristics are discovered" (Peikoff, 1979).

3. Individuals Have No Essences

Summary: Since the essence of a term is its connotation, and, as described earlier, Mill claims that individual terms have no connotation, it follows that "individuals have no essences."

Comments: This conclusion results from taking the narrower notion of connotation. If we take connotation to refer to *all* the properties of a thing, then clearly individual terms have a connotation. But do they have essences? The essence of an individual would be those characteristics that explain and make possible the largest number of the rest. In many cases we would find ourselves in agreement with the Schoolmen on the essence of an individual. For example, the essence of Julius Caesar is the same as the essence of man: rationality and animality.

4. Real Propositions, How Distinguished from Verbal

Summary: "An essential proposition, then, is one which purely verbal" and therefore "either gives no information or gives it respecting the name, not the thing. Non-essential, or accidental propositions, on the contrary, may be called real propositions, in opposition to verbal." This is because they predicate of a thing "some attribute not connoted by [its] name." Thus, they tell us a new fact that we didn't know before.

Comments: Although Mill's distinction between verbal and real propositions is fallacious, he has hinted at an important point. When we are told something we already know, we consider the proposition "merely verbal" since it tells us something that was already part of our understanding of the meaning of the subject. In this sense, it only unfolds the meaning of that term. Thus, a proposition is verbal *to me* if it expresses a part of the connotation of the term with which I was already familiar. In contrast, a real proposition is one that tells me something new, something that adds to my knowledge of the connotation of the subject.

In this sense 'Man has five fingers' is a *verbal* proposition to most people, since most people have known that men have five fingers almost as long as they have grasped the concept *man*. On the other hand 'Man is a rational animal' is for many people a *real* proposition the first time they hear it, since it tells them a matter of fact that they might not have previously considered: that the characteristic most distinctive of man and that explains the largest number of his other characteristics, is that he is rational.

5. Two Modes of Representing the Import of a Real Proposition

Summary: There are two different aspects in which a real, universal proposition can be considered. One "is best adopted to express the import of a proposition as a portion of our theoretical knowledge"; the other is useful "when the proposition is considered as a memorandum for practical use." In the first case the proposition 'All men are mortal' "means that the attributes of man are always accompanied by the attribute mortality." In the second case it "means that the attributes of man are *evidence* of, are a *mark* of, mortality. ..." The latter is usually the most useful view when we are studying the reasoning process.

Comments: We can express these notions symbolically as follows. That the attributes of man are always accompanied by the attribute mortal, means that mortal is one of the attributes of man, mortality $\in$ attributes(man). Alternately we can say that the connotation of *man* includes the connotation of mortal, connotation(man) $\supset$ connotation(mortal). The second interpretation says that when ever we find something that we can identify as *man*, we will know that that thing has the attribute *mortal*, man(x) $\rightarrow$ mortal(x).

7. On Classification and the Predicables

1. Classification, How Connected with Naming

Summary: Although the ideas of a *class* and *classification* play an important role in the work of many logicians, Mill does not stress these ideas. This is because he takes "general names as having a meaning, quite independently of their being the names of classes." Usually classes owe their existence to general names, since "[a]s soon as we employ a name to connote attributes, the things, be they more or fewer, which happen to possess those attributes, are constituted *ipso facto* a class. But in predicating the name we predicate only the attributes; and the fact of belonging to a class does not, in many cases, come into view at all." Killick (1909) illustrates this as follows: "Suppose I take two attributes, 'perfect molecular mobility' and 'inelasticity' and devise a name 'liquid', which shall connote or mean those properties, a class is *ipso facto* formed containing all objects possessing those two attributes."

Occasionally the opposite process takes place: classification precedes the formation of general name. This occurs when "we have thought it useful for the regulation of our mental operations, that a certain group of objects should be thought of together." Mill cites as an example the classes, orders, etc. of Cuvier's classification of plants and animals. Yet, even in these cases the resulting classes are, "as much as any other classes, constituted by certain common attributes, and their names are significant of those attributes, and nothing else."

Comments: In fact, classification usually precedes the definition of a general name in terms of essential attributes. For example, the meaning of 'liquid' is more than the two properties, 'perfect molecular mobility' and 'inelasticity.' Indeed we knew what liquids were long before we recognized that they are characterized by these two properties. And it is possible that in the future we might discover a substance having these two properties that we are not willing to classify as a liquid, because it is different from liquids in too many other respects.

We initially form a class on the basis of certain perceived similarities between the members of the class, that is, on the basis of certain common properties. However, the resulting general name connotes more than the properties that originally motivated the formation of the class. The general term denotes *all* of the individuals possessing the originally identified common attributes, and connotes *all* of the common properties of these individuals.

For example, the class of liquids is probably formed initially on the basis of certain unanalyzable resemblances. The denotation of 'liquid' is all liquids, and its connotation (in the wider sense) is all the properties of liquids. Among these, we may later discover, are perfect molecular mobility and inelasticity. If, within the context of our knowledge, these two properties seem the most characteristic of liquids, then we are justified, for now, in defining a liquid as an inelastic substance with perfect molecular mobility. We could then say that, within the context of our knowledge, these attributes are the *essential* attributes of liquids.

2. The Predicables

Summary: "The predicables are a fivefold division of General Names, not grounded as usual on a difference in their meaning, that is in the attribute which they connote, but on a difference in the kind of class which they denote." The predicables, handed down from Aristotle, are:

- Genus
- Species
- Differentia
- Property
- Accident

These distinctions do not apply to general names in isolation; they refer to all the ways that a name can be a predicate in a proposition. "The words genus, species, &c., are

therefore relative terms; they are names applied to certain predicates, to express the relation between them and some given subject ...."

3. Genus and Species

Summary: In popular usage *genus* and *species* are relative terms. Thus, "any two classes, one of which includes the whole of the other and more, may be called a Genus and Species." For example, *man* is a species of the genus *animal*, and *mathematician* is a species of the genus *man*.

The Aristotelian logicians used these terms in a more restricted sense. "It was requisite, according to their theory, that genus and species should be of the *essence* of the subject." Thus, *man* and *brute* could be considered coordinate species under the genus *animal*, but *biped* would not be considered a genus with respect to *man;* it would be considered a *property* or *accident* only. Does this distinction make any sense? The distinction is important, but the recourse to essence confuses the issue.

Comments: Full comments follow the next section.

4. Kinds Have a Real Existence in Nature

Summary: When we consider the classes denoted by general names, they seem to be of two kinds. "There are some classes, the things contained in which differ from other things only in certain particulars which may be numbered, while others differ in more than can be numbered, more even than we need ever expect to know." For example, the class of white things is distinguished by no common properties other than whiteness. "But a hundred generations have not exhausted the common properties of animals and of plants, of sulphur or of phosphorus; nor do we suppose them to be exhaustible ...."

"The differences [on which classification is based] are made by nature, in both cases, while the recognition of those differences as grounds of classification and of naming is, equally in both cases, the act of man ...." However, in the first case (e.g.,

white) the act of classification is motivated by convenience; in the second (e.g., *sulfur)* nature requires us to form the class: "the ends of language and of classification would be subverted if no notice were taken of the difference." Hence the latter classes are called *real kinds*, and the former *not-real kinds*.

The Aristotelian logicians considered only real kinds to be genera and species. In particular, the lowest real kind to which an individual is referable is called its species: the species of Isaac Newton is man. Although Newton belongs to many other classes (e.g., mathematician and Englishman) these are not species. The sexes, races, etc. would be considered not-real kinds if their differences turn out to be reducible to a few primary differences; if not, then they must be considered real kinds (i.e., species).

In summary, a real kind is a class "which is distinguished from all other classes by an indeterminate multitude of properties not derivable from one another." A real kind is a genus or species or both. A genus is a real kind which is divisible into other real kinds.

Comments: The crux of the problem with Mill's distinction between real and not-real kinds can be seen in the class *sulfur*. Mill calls this a real kind, since we will never exhaust the list of properties shared by all pieces of sulfur. On the other hand, it is quite possible that we may find that one property, say, its atomic structure, is the cause and explanation of all the rest. Then we would no longer be able to consider sulfur a real kind. Thus, if we can find a finite number of properties that explain all the rest (i.e., if we can *define* the class) then it is a not-real kind. A kind remains real only in so far as we are ignorant of it.

A class is initially demarcated on the basis of a finite number of properties. As we learn more about the class we continue to be aware of only a finite number of its properties. If all the known properties are explainable in terms of just a few, then we consider it a not-real kind. If this is not the case, and we suspect there is an "inexhaustible supply" of more unexplainable properties, then it is a real kind. By this reasoning

electron is not a real kind, because we believe all an electron's properties are explainable by a few quantum numbers. This illustrates the fact that the real/not-real distinction is at best an inessential one that is a reflection of our current knowledge of the class. It would probably be best to discard the distinction altogether.

5. Differentia

Summary: The word *differentia* "is correlative with the words genus and species, and ... signifies the attribute which distinguishes a given species from every other species of the same genus." Can we use any attribute that will distinguish the given species? The Aristotelian logicians say "No;" the differentia must be of the essence of the subject. Thus rationality could be considered the differentia of man; that he cooks his food could not be, since it is only an accidental property. Mill rejects this notion of differentia since it is based on the fallacious idea of *essence*. What then distinguishes the differentia from other properties? Mill observes that the connotation of a species includes the connotation of its genus. Therefore, since he has said that the essence of a name is just its connotation, he concludes that "the Differentia is that which must be added to the connotation of the genus, to complete the connotation of the species."

Comments: With regard to the notion that the differentia is added to the genus to yield the species, Joseph (1906) says, "Provided it is not supposed that the differentia is *added* to the common character of the 'larger class' in the same extraneous way that sugar is added to tea, there is no fresh harm in this mode of expressing oneself." But if the differentia is not an extraneous property added to the genus, then what is it? We are inclined to say, with the Aristotelians, that the differentia should in some way be essential.

If we take essence not in the medieval sense of an essential substance, but in the sense of those properties which cause or explain most of the rest, then the essence can be the source of the differentia. Joseph (1906) suggests the following criteria for the characteristics which form the differentia: "these characteristics should be (a) of the

same general kind for each type within one genus, or ... variations upon the same theme, in order to exhibit the mutual relations of agreement and divergence among the various types; (b) important, or ... pervasive: that is, they should connect themselves in as many ways as possible with the other characters of the species." These issues will be considered further when we discuss definition, in the next chapter.

6. Property

Summary: In the Aristotelian theory genus and differentia are of the essence of the subject. In contrast, *properties* and *accidents* are non-essential. The difference between properties and accidents is that properties follow *necessarily* from the essence, whereas accidents do not. Thus, that man uses language follows necessarily from his having a rational faculty, and is thus a property. That man has five fingers on each hand, follows neither from his rationality nor his animality, and is thus an accident.

"One attribute may follow from another in two ways; and there are consequently two kinds of [property]. It may follow as a conclusion follows premises, or it may follow as an effect follows a cause." An example of the former is the property of triangles that their angles sum to 180 degrees; an example of the latter is the property of man that he understands language.

Comments: Since the distinction between essential and nonessential attributes is a contextual distinction, the distinction between properties and accidents is also contextual. If we take the essence of a name to be those attributes which explain or cause most of the rest, then the properties are those attributes that the essence does explain, and the accidents are those attributes that it doesn't. Frequently an accident is an unexplained property. An accident may also be an attribute which, for reasons of cognitive economy, we have chosen not to include in the essence.

7. Accident

Summary: Accidents are divided into two classes, ***separable*** and ***inseparable***. Separable accidents are attributes that are universal to the species, but not necessary to it. For example, so far as we know, all crows are black, but this does not follow necessarily from crowness. And, if we discovered birds otherwise like crows but being white, we would call them 'white crows'; we would not say that they're not crows.

"Separable Accidents are those which are found, in point of fact, to be sometimes absent from the species; which are not only not necessary, but not even universal." For example being red-haired is a separable accident of man, since some, but not all, men are red-haired. Similarly, those attributes that are not even constant in an individual, such as to be sitting or walking, must be considered separable accidents.

8. Of Definition

1. A Definition, What

Summary: "The simplest and most correct notion of a definition is, a proposition declaratory of the meaning of a word, namely, either the meaning which it bears in common acceptation, or that which the speaker or writer, for the particular purposes of his discourse, intends to annex to it."

Notice that this means that words with no meaning, such as proper names, are not susceptible to definition. "In the case of connotative names, the meaning ... is the connotation, and the definition of a connotative name is the proposition which declares its connotation." This is most commonly done by predicating of the name intended to be defined "another name or names of known signification, which connote the same aggregation of attributes." Since *analysis* means the resolution of a "complex whole into the elements of which it is compounded," it can be seen that a definition is an analysis.

Comments: It is certainly counter-intuitive to say that proper names have no meaning, yet this follows from Mill's notion of connotation. As discussed earlier (Chapter 2, Section 4 and Chapter 6, Section 3), if we take connotation in the wider sense then we must conclude that individuals have a very rich connotation. Thus, as the connotation of a name grows, its extension narrows until the name becomes individual. Thus, as will be discussed below, proper names can be defined in the same ways as general names: by *ostension* (pointing) or by genus and differentia.

If we take connotation in the wide sense, as referring to all the properties possessed in common by the members of the denotation, then it is clear that a definition cannot cite all of the connotation. This would not be a definition. Rather, it would be a catalog of everything known about the subject, and it would still be incomplete. For a definition to serve the goals of cognitive economy it must condense our knowledge of the subject. Thus a definition should be in terms of the *essential* characteristics of the

thing defined. One way to accomplish this is to formulate the definition in terms of genus and differentia.

Several differences should be noted between this notion of definitions and that most common among modern logicians. The modern view is that a definition is merely a device for specifying the meaning of a word or phrase; it provides a sort of shorthand for a longer phrase. As a result, the word defined and its definition are completely interchangeable.

The view here is that the meaning of the name is probably already known; in any case the function of the definition is not to inform us of the meaning of the name. If we don't know what a liquid is, then being told that it is an incompressible substance with perfect molecular mobility, will not help much. Similarly, if we don't know the meaning of *man*, then being told the definition 'rational animal' won't be very useful; we still won't even know what a man looks like. Since a definition does not come close to exhausting the meaning of a name, it can be seen that a name and its definition are *not* interchangeable.

A correct definition results from a scientific analysis of the things named. The goal of this analysis is to determine the essential attributes of these things. Of course, as our scientific knowledge expands, we may find that in a wider context the attributes we thought were essential can no longer be considered so. We will then have to find attributes that are essential in this wider context. Thus, a definition is a kind of scientific law and must be validated like other scientific law.

2. What Names can be Defined?

Summary: "A name, ... whether concrete or abstract, admits of definition, provided we are able to analyze, that is, to distinguish into parts, the attribute or set of attributes which constitute the meaning both of the concrete name and of the corresponding abstract: if a set of attributes, by enumerating them; if a single attribute, by dissecting the fact or phenomenon (whether of perception or internal consciousness) which is

the foundation of the attribute." "The only names which are unsusceptible of definition, because their meaning is unsusceptible of analysis, are the names of the simple feelings themselves." For these "we are obliged to make a direct appeal to the personal experience of the individual whom we address."

3. Complete versus Incomplete Definitions

Summary: The only scientific definition of a name is one which declares the whole of the facts that the name involves in it signification. Most people do not want such a scientific definition however; they are seeking a guide to the correct use of a term. There are two sorts of these unscientific definitions. The first is an *essential but incomplete definition*, which defines a name on the basis of some but not all of the connotation of the name. An example is the definition, 'Man is the rational animal'. "Such definitions ... are always liable to be overthrown by the discovery of new objects in nature." For example, if we discovered a species of rational fish, we might have to revise our definition to 'Man is the rational mammal'.

Incomplete definitions are what the logicians had in mind when they said a definition should be *by genus and differentia.* The differentia is seldom all that is peculiar about the species; it is usually just one of many peculiar attributes.

Comments: In fact, almost all practical definitions are incomplete. As discussed previously; (Section 1) a definition does not simply define an abbreviation for a set of attributes; it specifies the *essential* attributes — those distinguishing attributes that best explain the other distinguishing attributes. Guidelines for the choice of a differentia have been discussed in the commentary accompanying Chapter 7, Section 5.

4. Complete Definitions versus Descriptions

Summary: The second kind of unscientific definition is the *accidental definition*, or *description*, which bases the definition of a name on something which is not part of the connotation at all, but still "enables us to discriminate the things denoted by the name

from all other things." "What would otherwise be a mere description may be raised to the rank of a real definition by the peculiar purpose which the speaker or writer has in view." In this case the author is in fact giving to some general name, "without altering its denotation, a special connotation," and then defining the name in terms of this special connotation. An example is Cuvier's definition of Man as "a mammiferous animal having two hands." "Scientific definitions, whether they are definitions of scientific terms, of of common terms used in a scientific sense, are almost always of [this] kind;" "their main purpose is to serve as landmarks of scientific classification. And since the classifications in any science are continually modified as scientific knowledge advances, the definitions in the sciences are also constantly varying."

Comments: This discussion, although based on Mill's incorrect idea of connotation, correctly points out the *contextual* nature of definitions: the correct definition of a term depends on the context in which that definition is to be used. This does not mean that definitions are arbitrary; rather, it means that the correctness of a definition is relative to a context. Within a given context of knowledge and purpose, a correct definition cites those properties that explain and make possible the greatest number of other attributes.

5. Real Definitions versus Nominal Definitions

Summary: Mill next turns to an ancient doctrine, that there are two kinds of definitions: definitions of names and definitions of things. "The former are intended to explain the meaning of a term; the latter, the nature of a thing, the last being incomparably the most important." Mill notes that the nominalist trend in his time was tending to discard the notion of real definitions (definitions of things) in favor of nominal definitions (definitions of names). [This trend has continued to the present.]

Mill takes all definitions to be nominal. "We apprehend that no definition is ever intended to 'explain and unfold the nature of a thing.' " His basis for this is that no one has ever been able to distinguish a definition of a thing from a proposition about that

thing. Hence he concludes that "All definitions are of names, and of names only ...." But in some definitions, "besides explaining the meaning of the word, it is intended to be implied that there exists a thing corresponding to the word." For example, the definitions of geometrical figures both give names to these figures and assert that they exist. If they didn't do the latter, it would be impossible to deduce true theorems from them. For example, when we say 'a triangle is a figure bounded by three straight lines', we are really making two statements in one package:

1. There exists a figure bounded by three straight lines.

2. We will call such a figure by the name 'triangle'.

Comments: I disagree with Mill on this point. If we consider a definition such as 'Man is the rational animal' it becomes apparent that if we hadn't previously known the meaning of 'man' then this definition wouldn't have been very useful. We still wouldn't know the most rudimentary things about men, such as their appearance. Rather, when we assert 'Man is the rational animal' we are generally assuming that the hearer knows the meaning of 'man'. This means that he is aware of a substantial part of its denotation and connotation. That is, he can recognize men when he encounters them, and he knows a number of the properties of men.

What, then, is the purpose of a definition such as 'Man is the rational animal'? Since it is not to inform us of the meaning of the word 'man', we can only conclude that it is to inform us of some fundamental fact about men. Thus against Mill I claim that, with a few trivial exceptions, there are no nominal definitions; the only definitions of interest in science and logic are *real* definitions.

Mill's objection to real definitions, that there is no way to distinguish a real definition from any other proposition asserting a property of the thing, can be answered as follows. He is right in the following sense: there is no way, once and forever, to pick out one or a few of an object's properties as the defining properties. This would imply

omniscience about the thing. As discussed previously, a real definition should be in terms of essential properties. But which properties are essential and which are not is a contextual issue: it must be settled relative to the available knowledge and the intended use of the definition.

6. Mathematical Definitions

Summary: What is the meaning of a mathematical definition? Consider a definition such as 'A circle is a plane figure bounded by a line all the points of which are at an equal distance from a given point within it'. This cannot be considered an assertion about real circles, since in no real circle are the radii exactly equal. Some people have said "that the subject-matter of mathematics, and of every other demonstrative science, is not things as they really exist, but abstractions of the mind. A geometrical line is a line without breadth; but no such line exists in nature; it is a notion merely suggested to the mind by its experiences of nature."

Mill disagrees with this view, since he claims the mind "cannot conceive length without breadth; it can only, in contemplating objects, attend to their length, exclusively of their other sensible qualities, and so determine what properties may be predicated of them in virtue of their length alone." Thus, "the postulate involved in the geometrical definition of a line is the real existence, not of length without breadth, but merely of length, that is, of long objects."

Comments: Another way of saying this is that a line is a thing whose breadth is negligible. When is a thing's breadth negligible? That depends on circumstances; for some purposes a thing's breadth will be negligible, while for others it won't. Thus, whether a real thing is considered a line or not is a contextual issue. This is the case with most mathematical concepts. All mathematical concepts are models of the real world that attend to the significant aspects, and ignore the negligible aspects, of some real phenomena. Indeed, mathematics can be defined as the *systematic ignoring of the negligible.*

7. Definitions Grounded on Knowledge of Corresponding Things

Summary: "Although, according to the opinion here, definitions are properly of names only, and not of things, it does not follow from this that definitions are arbitrary. How to define a name, may not only be an inquiry of considerable difficulty and intricacy, but may involve considerations going deep into the nature of the things which are denoted by the name." These inquiries are not "so much to determine what is, as what should be, the meaning of a name ...."

It often happens that names are applied to things solely on the basis of resemblance; the names have no distinct connotations in the minds of their users. "This, as we have seen, is the law which even the mind of the philosopher must follow, in giving names to the simple elementary feelings of our nature; but, where the things to be named are complex wholes, a philosopher is not content with noticing a general resemblance; he examines what the resemblance consists in[,] and he only gives the same name to things which resemble one another in the same definite particulars. The philosopher, therefore, habitually employs his general names with a definite connotation." To accomplish this requires an inquiry into matters of fact.

"In giving a distinct connotation to the general name, the philosopher will endeavor to fix upon such attributes as, while they are common to all the things usually denoted by the name, are also of greatest importance in themselves; either directly, or from the number, the conspicuousness, or the interesting character, of the consequences to which they lead." He will select those *differentiae* that lead to the greatest number of interesting properties. But this may be "one of the most difficult of scientific problems" —and one of the most important. "[S]ome of the most profound and most valuable investigations" in philosophy have "offered themselves under the guise of ... inquiries into the definition of a name."

II. REFERENCES

Aristotle. *Categories, On Interpretation, and Prior Analytics* (trans. H. P. Cooke). Loeb Classical Library, Cambridge: Harvard University Press, 1973.

Creighton, J. E. *An Introductory Logic*. New York: MacMillan, 1906.

De Morgan, A. *Formal Logic: or, The Calculus of Inference, Necessary and Probable*. London: Taylor and Walton, 1847.

Flew, A. *A Dictionary of Philosophy*. New York: St. Martin's Press, 1979.

Frank, P. G. *The Validation of Scientific Theories*. New York: Collier Books, 1954.

Hempel, C. G. A Logical Appraisal of Operationism, in Frank (1954).

Jevons, W. S. *Elementary Lessons in Logic: Deductive and Inductive*. New York: MacMillan, 1919.

Joseph, H. W. B. *An Introduction to Logic*. Oxford: Clarendon Press, 1906, second edition.

Killick, A. H. *The Student's Handbook Synoptical and Explanatory of Mr. J. S. Mill's System of Logic*. London: Longmans, Green, and Co., 1909.

Mill, J. S. *A System of Logic, Ratiocinative and Inductive, being a connected view of the principles of evidence and the methods of scientific investigation*. London: Longmans, Green, and Co., 1843, eighth edition.

Mill, J. S. *John Stuart Mill's Philosophy of Scientific Method*, edited with an introduction by Ernest Nagel, New York: Hafner, 1950.

Nozick, R. *Philosophical Explanations*. Cambridge: The Belknap Press of Harvard University Press, 1981.

Peikoff, L. The Analytic-Synthetic Dichotomy, in Rand (1979).

Quine, W. V. *From a Logical Point of View*. Cambridge: 1953.

Rand, A. *Introduction to Objectivist Epistemology*. New York: New American Library, 1979.

Whately, R. *Elements of Logic*. London: Longmans, Green, Reader and Dyer, 1868, ninth edition.

INITIAL DISTRIBUTION LIST

Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Dudley Knox Library Code 0142 Naval Postgraduate School Monterey, CA 93943	2
Office of Research Administration Code 012A Naval Postgraduate School Monterey, CA 93943	1
Chairman, Code 52Hq Department of Computer Science Naval Postgraduate School Monterey, CA 93943	40
Professor Bruce J. MacLennan, Code 52Ml Department of Computer Science Naval Postgraduate School Monterey, CA 93943	12
Dr. Robert Grafton Code 433 Office of Naval Research 800 N. Quincy Arlington, VA 22217	1
A. Dain Samples Computer Science Division - EECS University of California at Berkeley Berkeley, CA 94720	1
Professor Douglas Smith, Code 52Sc Computer Science Department Naval Postgraduate School Monterey, CA 93943	1

DUDLEY KNOX LIBRARY
3 2768 00337507 2